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Hybrid experimental/numerical technique for determination of the complex dynamic moduli of elastic porous materials †

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Abstract

Polyurethane (PU) and other plastic foams are widely used as passive acoustic absorbers. For optimal design, it is often necessary to know the viscoelastic properties of these materials in the frequency range relevant to their application. An experimental/numerical technique has been implemented to determine the Young and shear dynamic moduli and loss factor of poroelastic materials under low-frequency 40-520Hz random excitation. The method consists of measuring the dynamic response of the sample at its surface, and matching the response with the predictions from a finite element model in which the two complex elastic moduli are the adjustable parameters. Results are presented for measurements made in air, under standard pressure and temperature conditions, and compared with predictions based on Okuno's model. The dependence of elastic moduli on the dimension of the sample and its boundary conditions is also studied.

Keywords: Young's modulus; Shear modulus; Loss factor; Finite element method

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1. Introduction

For many years, porous materials such as polyurethane (PU) foams have been widely used for passive sound absorption and noise control in automotive and aircraft applications. At low frequencies, the porous medium is characterized by geometrical parameters such as porosity, flow resistivity, tortuosity and by the mechanical parameters of the frame [1]. When the skeleton is set in motion, the dynamic behavior is described by Biot's theory [2] where the viscoelastic properties of the solid phase are involved in a wide frequency range.

In order to use these materials effectively, it is critical to determine accurately their complex, Young and shear moduli. Several techniques exist for the experimental determination of frequency dependent moduli of viscoelastic solids [3,4]. These techniques can be broadly classified as resonant and non-resonant and consist generally in measurements on a narrow frequency range associated with measurements at different temperatures. A simple experimental method in the resonance technique is proposed by Madigosky and Lee [5,6] in which a longitudinal wave is transmitted down a rod of material with accelerometers attached at both ends. The complex modulus of elasticity can be calculated at each resonance frequency from the phase interference of the extensional waves in the bar. The technique works throughout the audio range but is limited to a specific one-dimensional sample geometry, a discrete set of resonance frequencies, and it measures only the complex Young's modulus. A similar technique has been developed by Garrett [7] to measure both the Young's and shear moduli in a rod in which torsional, longitudinal, and flexural resonant modes are selectively excited by a pair of coils placed

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at the end of the rod in a magnetic field. Again, the technique is limited to one-dimensional geometry and a discrete set of frequencies. Also, the above-mentioned techniques are not suitable for elastic porous materials as these methods do not take into account the three wave types (two longitudinal and one transverse) which have been shown to simultaneously contribute significantly to the observed acoustical behavior of elastic porous materials such as foams [2]. A system developed by Polymer Laboratories (now Rheometric Scientific, NJ) is the dynamic mechanical thermal analyzer (DMTA) [8]. This elaborate system automatically measures Young's modulus as a function of temperature, in a small sample, over a limited range of frequency, typically 0.3–30Hz. The master curve covering the entire frequency range is then obtained by using the time-temperature superposition principle and the associated WLF shift constant. This principle states that measurements made within a narrow frequency range at several temperatures are equivalent to measurements made over a wide frequency range at a single temperature. Willis et al. [9] proposed a method for measuring the two complex elastic moduli simultaneously. The method consists of measuring the dynamic response of a sample with a set of five independent laser interferometers, and matching the response with the predictions from a finite element code.

We propose a direct method for measuring the complex elastic moduli, with a sample of arbitrary shape, over a continuous frequency range. The method consists of measuring the dynamic response of sample under tension-compression and shear modes separately. We match this response with the prediction from a two-dimensional elastic-absorption finite element model developed by Kang and Bolton [10] and then use an inversion scheme to obtain the dynamic complex Young and shear moduli. The results are compared with the technique developed by Okuno [11]. The effects of the varying dimensions in the lateral direction and different boundary conditions of the sample on these complex moduli are also explored. The study will be useful to help determine the optimal dimensions of the sample to measure the dynamic moduli for practical purposes. The experimental/numerical technique is presented in Sec. 2, the results are shown and discussed in Sec. 3, and concluded in Sec. 4.

2. Hybrid experimental/numerical technique

2.1 Experimental setup

Two types of tests were undertaken in order to provide experimental information. These tests are the dynamic response measurements of a slab of foam in tension-compression, and two slabs of foam in shear. The tension-compression tests were carried out in order to investigate the dynamic poroelastic material properties of the foams. Different widths of specimen were prepared in order to study the effects of flow path length on the damping capacity of the foams. The shear tests were performed to investigate the dynamic properties of the solid matrix of foams, since there are theoretically no fluid effects in shear deformation.

The apparatuses used for measuring the Young's modulus and shear modulus were suggested by Okuno [11] and are sketched in Fig. 1. The basic operating mechanism is exactly the same in both cases. Test specimen was attached to support plates, which were made of aluminum. The lower plate was fixed to a stationary bed. The upper plate was connected to an electromagnetic shaker by a connecting rod (stinger) via a piezoelectric force transducer. An accelerometer was attached to the upper plate by bee's wax. The upper plate, and hence the sample, was excited by the electromagnetic shaker through the stinger and the force transducer. The key to making precise measurements of the dynamic response of the sample is to ensure the aluminum plate makes good contact with the testing sample, but does not statically compress the sample.

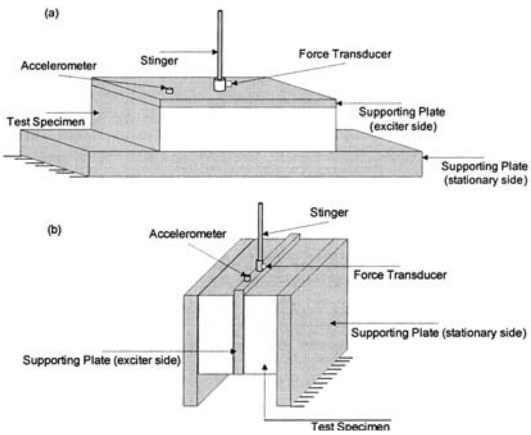

Fig. 1. Test arrangement for experiment in (a) tension compression and (b) shear.

In the case of shear mode, instead of using one single sample piece, an aluminum plate was sandwiched between two nearly identical samples to preserve symmetry, and therefore obtain a pure vertical plate movement. The center plate was excited by the shaker through a stinger and force transducer.

A random excitation was used for both the tension-compression and shear test. The force and accelerometer signals were fed to an FFT (fast Fourier transform) analyzer and transfer functions were calculated. The transfer function H(ω) is expressed as

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \tag{1}$$

where $X(\omega)$ and $Y(\omega)$ are the Fourier transforms of the input (force) and output (acceleration) signals. To minimize the inaccuracy in the measurement of surface acceleration caused by the rocking effect due to the vibration of the stinger connecting the shaker and the supporting plate on the exciter side, an average of the measurements was taken for three different positions of the accelerometer on the plate. A schematic of the experimental procedure is shown in Fig. 2.

The tension compression test was intended to be carried out by using two different sets of boundary conditions for the sides of the test specimen. In the first case, the sides of the sample were left unconstrained to get free boundary conditions, while in the other instance a lubricated boundary condition was introduced for the sides of the sample. This was achieved by fixing the sample in a box of acrylic made of exactly the same dimensions as that of the sample. In the shear test all the side faces of the specimen were open to the air.

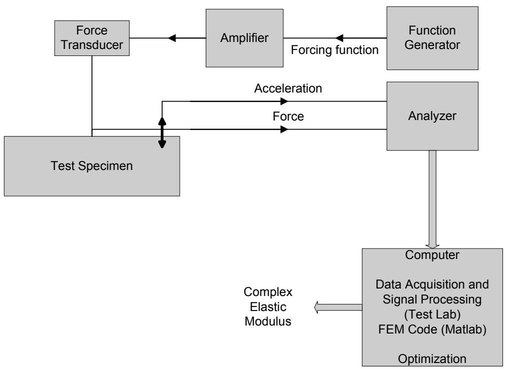

Fig. 2. Schematic of Experimental/numerical procedure to determine the complex elastic moduli.

2.2 Finite element model

Elastic-absorption foam finite element model by Kang and Bolton has been used, which itself is based on the elastic porous material theory of Shiau and Bolton [10,12]. Both these models have been derived from Biot's dynamic theory of poroelasticity [2]. It differs from the approach of Okuno, as in Biot's theory the effects of both fluid inertia and dissipation are accounted for. Okuno, on the other hand, uses the quasi-static theory where the effect of inertia is neglected and the dissipation is linearly dependent on the flow velocity (Darcy's Law) [11].

Elastic porous materials are generally characterized in terms of seven or nine macroscopically measurable physical properties of the solid and fluid phases, including flow resistivity, porosity, tortuosity, the bulk density, the in vacuo bulk Young's modulus, the associated loss factor, the Poisson's ratio, VCL (viscous characteristic length), and TCL (thermal characteristic length). The values of the properties needed to model the test specimen are shown in Table 1. Some of them are, in principle, measurable while the others need to be determined by matching theoretical and experimental results. The detailed procedures are well described by Kang and Jung [13].

The size of the elements in this finite element model varied with the variation in width of the sample. Kang and Bolton [10] have already shown that for an analysis of up to 2 kHz, the finite element foam model is accurate for acoustic and foam elements of size equal or below 27mm×27mm. For lower frequencies, even larger sized elements can be used. In our experiment, the height of each sample was constant at 30mm and the element size along the height is maintained at 2.3mm. The widths of the samples, on the other hand, are varying and are shown in Table 2. The number of nodes along the width of the sample is 13 for all the cases. So the distance between two nodes varies from 6.2mm (in case of 75mm wide sample) to 16.6mm (for 200mm wide sample). Thus, in our finite element model the element size varies from 6.2mm×2.3mm to 16.6mm×2.3mm.

Two sets of boundary conditions, lubricated and free, corresponding to the experimental conditions, were applied to predict the transfer function of the sample in tension compression mode, while a separate model to simulate the sample in shear was also developed.

Table 1. Elastic Porous Material Properties used in calculations.

Bulk Density [Kg/m^3]	42
Thickness [mm]	30
Porosity	0.98
Flow Resistivity [MKS Rayls/m]	13330
Structure Factor	1.48
Poisson's Ratio	0.34
VCL [μm]	108.10
TCL [μm]	195.72

Table 2. Cross-sectional dimensions of the samples used in experiment.

Sample	Dimensions [mm^2]
1	75 × 75
2	100 × 100
3	125 × 125
4	150 × 150
5	175 × 175
6	200 × 200

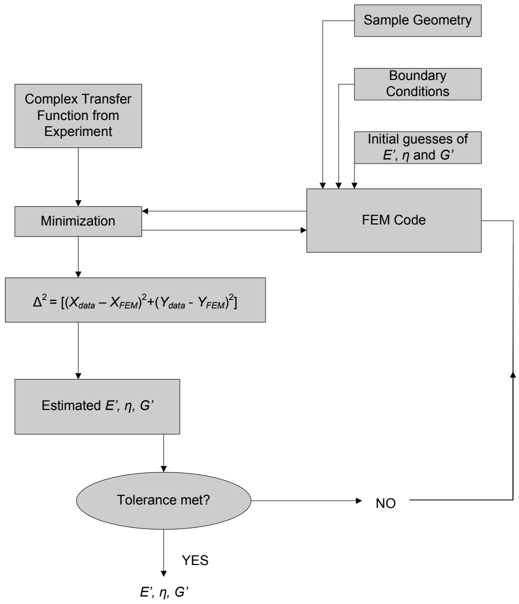

Fig. 3. Block diagram of the experimental/numerical procedure for determining the material properties.

2.3 Numerical inversion

This procedure is used to couple the data obtained from experiment with the predictions made by FEM. The algorithm used to extract the material properties from the data consists of minimizing the difference between the data and the predicted values. The finite element code is written in terms of two material parameters: Young's modulus, E' (or alternately the shear modulus G'), and the loss factor, η. As usual, the complex Young's modulus is defined as $E = E'(1+i\eta)$. Similarly, in the shear mode of the experiment, the complex shear modulus is defined as $G = G'(1+i\eta)$. The values of these parameters (E'/G', η) are obtained by a two-dimensional direction set method (Powell's method) [14], a robust, classical method in optimization theory. The method requires initial estimates of both the parameters and a step size for the search procedure. These initial guesses can be obtained from either static tests or by using an already established theory like Okuno's. After the completion of optimization at first step, the results can be used as the initial guess for the next frequency step. The function that is minimized is the mean-square error defined by

$$\Delta^2 = [(X_{data} - X_{FEM})^2 + (Y_{data} - Y_{FEM})^2] \quad (2)$$

where Xdata, Ydata are the real and imaginary parts, respectively, of the complex amplitude of the transfer function from the measured data. Similarly XFEM, YFEM are the real and imaginary parts of the transfer function obtained from the FEM prediction. The algorithm converges toward the optimum values of *E'*, η and *G'*, η for the tension-compression and shear modes respectively. The process is repeated at each frequency over the range of interest. A block diagram of this procedure is shown in Fig. 3.

3. Results and discussion

3.1 Dependence of Young's modulus on sample size and boundary conditions

Before undertaking the experimental procedure, predictions using only the finite element method were made for the surface dynamics of the sample with constant Young's modulus and varying dimensions. It was found that the surface velocities for a given constant force decrease with an increase in the width of the sample. The difference between velocities with different dimensions continuously declines as the width of the sample increases. At around 200mm width of the sample, further increasing the width has almost no effect on the surface dynamics of the sample as shown in Fig. 4(a). This is the case when the boundaries on both sides of the sample are free.

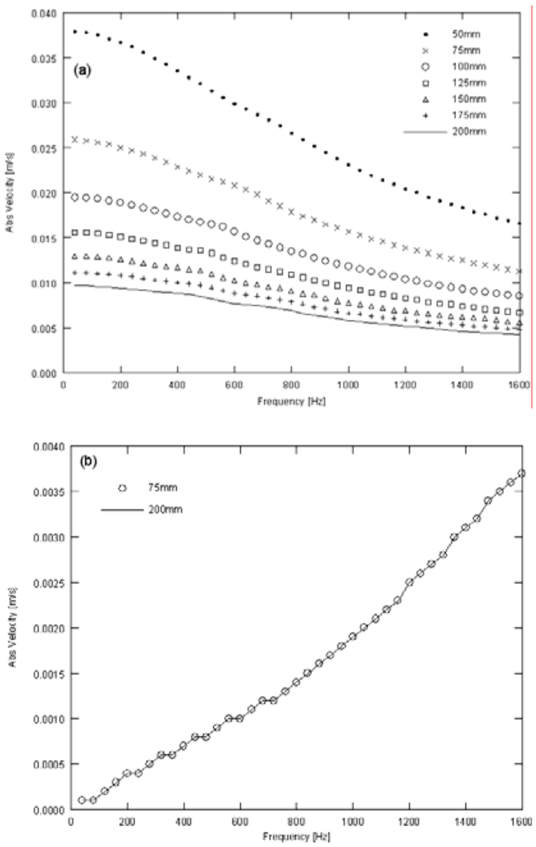

Fig. 4. Predicted dynamic response by FEM with constant Young's modulus for different dimensions of samples: (a) free boundary conditions (b) lubricated boundary conditions.

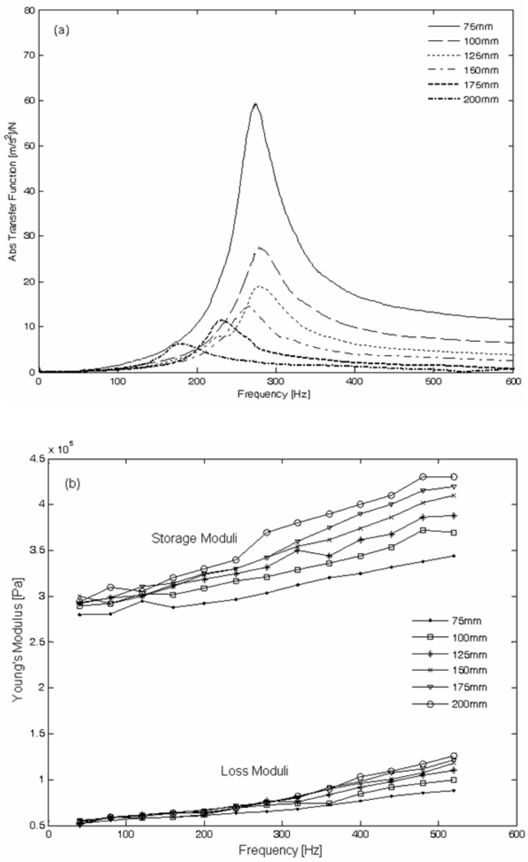

Fig. 5. Measurements with sample in tension compression (a) Transfer function, (b) Young's modulus.

Fig. 4(b) shows that in case of lubricated boundary conditions, the response of the sample is similar to varying sample dimensions. This is very understandable because when lubricated boundary conditions are applied to the sides, we are allowing the sample to slide along the boundary but the motion against the boundary is constrained, so in the lateral direction, the sample achieves the condition of infinite length and hence its dynamic response becomes independent of varying dimensions in this direction. The varying thickness of the sample, in this case, should change the dynamic response, but as the samples used in our experiments and simulations have uniform thickness, a consistent dynamic behavior of the sample is achieved. Experiments were conducted for the samples with six different dimensions in the case of free boundary conditions on the sides, as stated in Table 2. The natural frequency of the supporting plate restricted our measurements to a frequency of 520 Hz to avoid any undue interference in the dynamic response of the samples. The material of the test specimen was identical for all the experiments and its properties are listed in Table 1. First, the transfer functions (acceleration/force) were measured for all the samples by using the experimental arrangement in tension compression mode. The magnitude of these transfer functions is shown in Fig. 5(a). The Young's storage and loss moduli corresponding to these transfer functions are shown in Fig 5(b). It is clear that, as predicted, an increase in the dimension of the sample causes a decrease in the dynamic response of the sample and Young's modulus of the sample experiences an increase. In Fig. 6, a comparison between the storage and loss moduli, obtained using the current method and Okuno's method, is presented for the samples with widths of 100mm and 200mm, respectively. The results used for comparison are taken from the experimental method of Okuno. The response functions derived by Okuno in tension compression mode and shear mode are

$$\frac{\ddot{W}}{F} = \frac{-\Omega^2}{\frac{A_1}{h}\tilde{E} - m_1\Omega^2} \quad (3)$$

$$\frac{\ddot{W}}{F} = \frac{-\Omega^2}{\frac{2A_2}{h}\tilde{G} - m_2\Omega^2} \quad (4)$$

where $\ddot{W}$ and F are the acceleration and force at the supporting plate, A1 is the cross-sectional area of the specimen , m1 is the mass of the supporting plate, h is the thickness of the specimen. $\tilde{E}$ and $\tilde{G}$ are the complex Young's and Shear moduli of the specimen. It is noteworthy that as the dimensions of the sample increase, the agreement between the two sets of results decreases. This is due to the fact that the natural frequency of the supporting plate at the exciter side in the experiment decreases as its size increases and, as a result, has a direct effect on the dynamic behavior of the sample.

Experiments were also done with lubricated boundary conditions for the samples of width 100mm and 200mm and the resulting Young's storage and dynamic moduli are shown in Fig. 7. The results show that there is no significant effect of the width of the sample when this type of boundary is applied to it.

3.2 Shear modulus and poisson's ratio

Next we present the results of the shear storage and loss moduli for the samples of width 100mm and 125mm in Fig. 8 (a). The experimental apparatus employed in this study limited the dimension of the sample for the shear test to a maximum of 125mm, so more tests could not be performed. One way to verify this result would be to plot the real part of Young's modulus as a function of the real part of the shear modulus. Ideally, there should be a linear relationship between the two, and Fig. 8 (b) shows that it is nearly so. The standard relation between E' and G' involves Poisson's ratio, ν:

$$\frac{G'}{3G' - E'} = \frac{1}{1 - 2\nu} \quad (5)$$

The loss factors of the bulk modulus and shear modulus are generally identical, so that the ratio of sound speeds and Poisson's ratio are real. Physically, it means that all the losses are due to the conversion

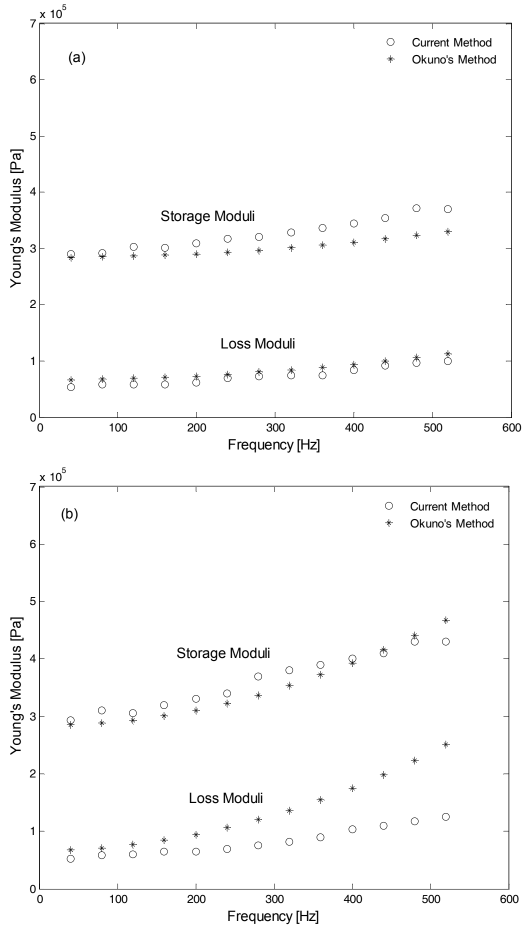

Fig. 6. Measurements with sample in tension compression (a) Transfer function, (b) Young's modulus.

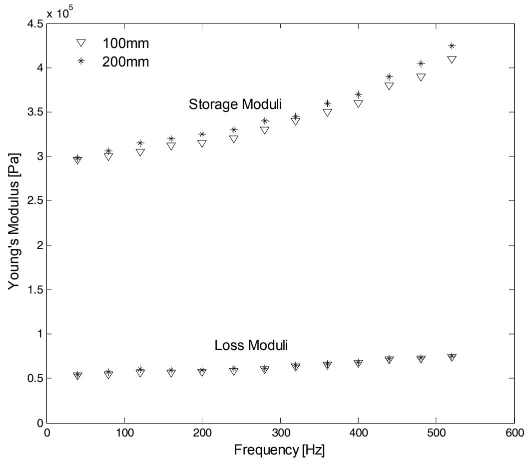

Fig. 7. Young's modulus for samples with lubricated boundaries on sides.

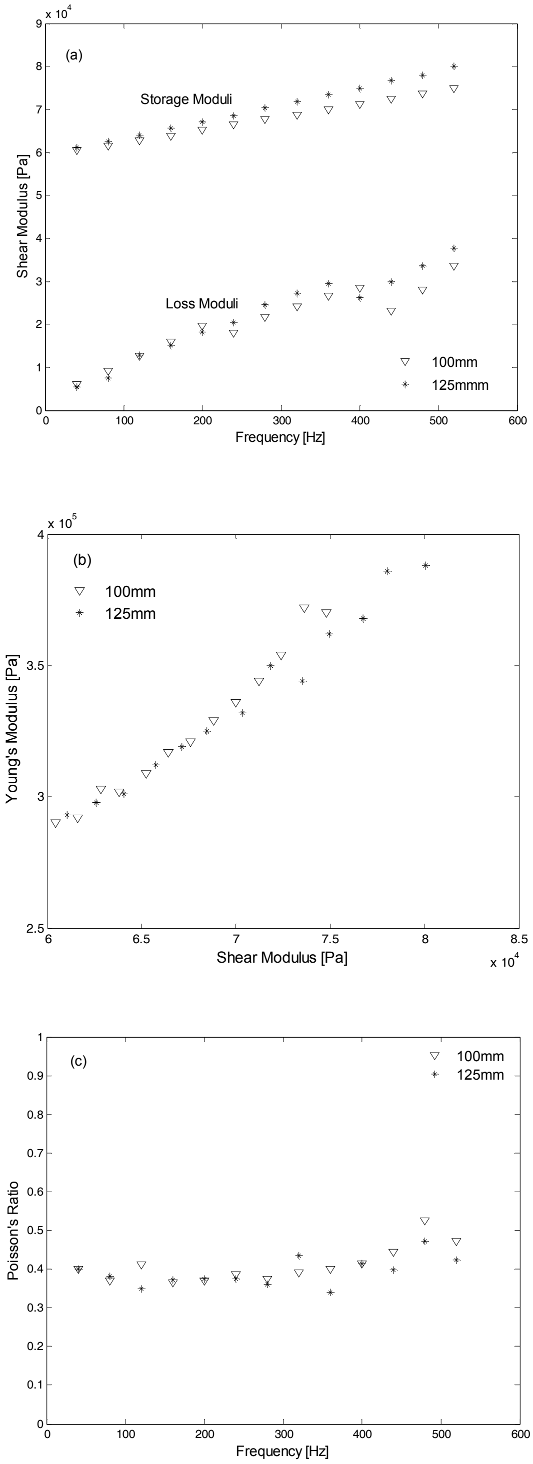

Fig. 8. Measurements with sample in shear (a) Shear modulus, (b) Young's modulus as a function of shear modulus, (c) Poisson's ratio

of dilatational to shear energy at the boundaries of the air inclusions, with a corresponding dissipation of the shear waves into heat in the material as explained by Jarzynski [15]. In Fig. 8(c), Poisson's ratio of the test material is plotted as function of frequency by using the values of Young's and shear dynamic moduli and is found to be virtually frequency independent.

4. Conclusions

The minimum suitable dimensions required to measure the dynamic properties of elastic porous materials for practical applications were analytically determined to be around 200mm×200mm. The results of the hybrid experimental/numerical procedure show that Young's modulus of an elastic porous material increases as the dimensions of the sample are increased with free boundary conditions on the sides of the sample. Alternatively, if lubricated boundary conditions are applied, the change in dimensions does not affect the value of the dynamic elastic moduli of the material significantly. Shear modulus was found to have a linear relationship with Young's modulus. We also establish from this study that Poisson's ratio for the foam material is frequency independent to a great extent.

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